LOW-COST ADAPTIVE MAXIMUM ENTROPY COVARIANCE MATRIX RECONSTRUCTION FOR ROBUST BEAMFORMING

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ABSTRACT

In this paper, we present a novel low-complexity adaptive beamforming technique using a conjugate gradient (CG) algorithm to avoid matrix inversions. The proposed method exploits algorithms based on the maximum entropy power spectrum (MEPS) to estimate the noise-plus-interference covariance matrix (MEPS-NPIC) so that the beamforming weights are updated adaptively, thus greatly reducing the computational complexity. MEPS is further used to reconstruct the desired signal covariance matrix and to improve the estimate of the desired signals's steering vector (SV). Simulations show the superiority of the proposed MEPS-NPIC approach over previously proposed beamformers.

Index Terms— Adaptive beamforming, Conjugate gradient, Matrix reconstruction, Spatial power spectrum.

1. INTRODUCTION

Adaptive beamforming aims to extract the signal from a certain direction while suppressing interference and noise. The technique has been widely used in many fields such as wireless communications, sonar and radar [1]. However, standard adaptive beamformers are well known to be sensitive to steering vector (SV) mismatches, array imperfections or environmental uncertainties due to non-ideal conditions and many different factors (e.g., wavefront errors, local scattering and finite sample sizes) [2]. Hence, various adaptive beamformers have been developed in order to mitigate the effects of these problems. Existing methods include diagonal loading [3, 4], the worst-case optimization in [5] and the projection techniques and eigenspace-based beamformer investigated in [6–9]. However, these approaches are limited when the SV mismatch is severe.

Recent works show that the main cause of beamformer performance degradation is the leakage of a component of the signal of interest (SOI) in the sample covariance matrix (SCM) [10]. Recently, an approach to adaptive beamforming was presented that removes the effect of the SOI component from the covariance matrix by reconstructing the noise-plus-interference covariance (NPIC) matrix. The NPIC matrix in [11] is reconstructed by integrating the assumed SV and the corresponding Capon spectrum over a range of angles in which the SOI is absent. The NPIC-based beamformer in [12] relies on a sparse reconstruction method. In [13, 14], computationally efficient algorithms via low complexity reconstruction of the NPIC matrix are presented. In [15] a subspace-based NPIC matrix reconstruction algorithm was proposed. Later, in [16] an approach is developed using spatial power spectrum sampling (SPSS). In [17] the SOI component is eliminated from the direction of arrival (DoA) of the related bases in order to construct an NPIC matrix directly from the signal-interference subspace.

It is worth noting that the use of adaptive antenna arrays and their applications has a trade-off between computational complexity and performance which has a direct relation with the adaptation algorithm. However, in practice and for large systems, these techniques require the computation of the inverse of the input data SCM (or NPIC matrix), rendering the method very complex.

In this work, we develop a conjugate gradient (CG) adaptive version of the maximum entropy power spectrum noise-plus-interference covariance (MEPS-NPIC) technique in [18], denoted MEPS-NPIC-CG. The proposed MEPS-NPIC-CG algorithm updates the beamforming weights with a reduced cost as it does not explicitly form the covariance matrices, relying instead on low-cost iterative techniques. The estimated weight vector is obtained from a coarse estimate of the angular sector where the desired signal (DS) lies, using CG iterations that avoid the explicit construction of the covariance matrix. We similarly implicitly reconstruct the DS covariance matrix and obtain a better desired signal SV estimate using low-cost iterations. An analysis of computational complexity shows that MEPS-NPIC-CG has low-complexity and outperforms other existing techniques.

2. PROBLEM BACKGROUND

Let us assume a linear antenna array with M sensors, spaced by distance d, that receive narrowband signals which impinge on the array from several far-field sources. The array observation vector at the t-th snapshot can be modeled as

$$\mathbf{x}(t) = \mathbf{a}(\theta_0)s_0(t) + \sum_{l=1}^{L} \mathbf{a}(\theta_l)s_l(t) + \mathbf{n}(t), \qquad (1)$$

where $s_0(t)$ and $s_l(t)$ denote the waveforms of the SOI and lth interfering signal, respectively. The additive white Gaussian noise vector $\mathbf{n}(t)$ is assumed spatially uncorrelated from the DS and the interfering signals. The angles θ_0 and θ_l ($l = 1, \cdots, L$) denote the DoAs of the DS and interference, respectively. For a sensor array with M sensors, $\mathbf{a}(\cdot)$ designates the corresponding SV, which has the general form $\mathbf{a}(\theta) = \left[1, e^{-j\pi\bar{d}\sin\theta}, \cdots, e^{-j\pi(M-1)\bar{d}\sin\theta}\right]^{\mathrm{T}}$, where $\bar{d} = 2d/\lambda$, λ is the wavelength, and $(\cdot)^{\mathrm{T}}$ denotes the transpose. Assuming that the SV $\mathbf{a}_0 = \mathbf{a}(\theta_0)$ is known, for a given beamformer $\mathbf{w}$, the performance is evaluated by the output signal-to-interference-plus-noise ratio (SINR) as

$$\mathrm{SINR} = \sigma_0^2|\mathbf{w}^{\mathrm{H}}\mathbf{a}_0|^2/\mathbf{w}^{\mathrm{H}}\mathbf{R}_{\mathrm{i+n}}\mathbf{w}, \tag{2}$$

where $\mathbf{R}_{\mathrm{i+n}}$ is the NPIC matrix, σ_0^2 is power of the DS and $(\cdot)^{\mathrm{H}}$ stands for Hermitian transpose. The beamformer that maximizes (2) is equivalent to finding the solution that maintains a distortionless response toward the SV $\mathbf{a}_0$:

$$\min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}}\mathbf{R}_{\mathrm{i+n}}\,\mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^{\mathrm{H}}\mathbf{a}_0 = 1. \tag{3}$$

The solution to (3) yields the optimal beamformer given by

$$\mathbf{w}_{\mathrm{opt}} = \mathbf{R}_{\mathrm{i+n}}^{-1}\mathbf{a}_0/\mathbf{a}_0^{\mathrm{H}}\mathbf{R}_{\mathrm{i+n}}^{-1}\mathbf{a}_0, \tag{4}$$

which is the adaptive weight vector based on the minimum variance distortionless response (MVDR) criterion [1]. Moreover, the array covariance matrix $\mathbf{R} = \mathrm{E}\{\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)\}$ is

$$\mathbf{R} = \mathbf{R}_{\mathrm{i+n}} + \mathbf{R}_{\mathrm{s}} = \int_{\Phi} P(\theta)\mathbf{a}(\theta)\mathbf{a}^{\mathrm{H}}(\theta)d\theta, \tag{5}$$

where $P(\theta)$ is the power spectrum of the signals and $\Phi = [\bar{\Theta} \cup \Theta]$ covers the union of the angular sectors of the noise-plus-interference signal, $\bar{\Theta}$, and of the DS region, Θ (obtained through some low-resolution direction finding methods [1]), while $\mathbf{R}_{\mathrm{s}} = \sigma_0^2\mathbf{a}_0\mathbf{a}_0^{\mathrm{H}}$ is the theoretical DS covariance matrix. Since $\mathbf{R}_{\mathrm{i+n}}$ is unknown in practice, it is substituted by the data SCM as $\hat{\mathbf{R}} = (1/K)\sum_{t=1}^{K}\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)$, where K is the number of received snapshot vectors.

3. PROPOSED MEPS-NPIC-CG ALGORITHM

3.1. Maximum Entropy Power Spectrum

In the proposed beamforming method, an approach different from prior works is adopted to reconstruct the NPIC and the DS covariance matrices. The essence of the idea is based on the use of the spatial spectrum distribution over all possible directions and coarse estimates of the angular regions where the DS and the interferers lie. In this work, we exploit maximum entropy power spectrum estimation [19]:

$$\hat{P}_{\mathrm{meps}} = \frac{1}{\epsilon_p|\mathbf{a}^{\mathrm{H}}(\theta)\hat{\mathbf{R}}^{-1}\mathbf{u}_1|^2} \tag{6}$$

where $\mathbf{u}_1 = [\,1\ 0\ \cdots\ 0\,]^{\mathrm{T}}$, $\epsilon_p = 1/\mathbf{u}_1^{\mathrm{T}}\hat{\mathbf{R}}^{-1}\mathbf{u}_1$.

3.2. Desired Signal SV Estimation

In practice, we have inaccurate SV estimates, resulting in performance degradation. Therefore, we utilize the knowledge of the angular sector of the SOI to construct a criterion which can be used to estimate the actual SV. This algorithm is based on the multiplication of an estimate of the DS covariance matrix and the nominal SV of the SOI, which results in a vector much closer to the SV of SOI. First, the DS covariance matrix can be reconstructed based on MEPS by numerically evaluating (5) over Θ

$$\hat{\mathbf{R}}_{\mathrm{s}} \approx \sum_{i=1}^{S} \hat{P}_{\mathrm{meps}}\,\mathbf{a}(\theta_{s_i})\mathbf{a}^{\mathrm{H}}(\theta_{s_i})\Delta\theta_s, \tag{7}$$

where Θ is sampled uniformly with S sampling points spaced by $\Delta\theta_s$, so that $\{\mathbf{a}(\theta_{s_i})|\theta_{s_i} \in \Theta\}$ lies within the range space of $\hat{\mathbf{R}}_{\mathrm{s}}$. Let $\bar{\mathbf{a}}$ be the nominal desired signal SV. If the set Θ is such that the noise and interference power are dominated by the signal power in the covariance estimate (7), then

$$\hat{\mathbf{a}}_0 = \hat{\mathbf{R}}_{\mathrm{s}}\bar{\mathbf{a}} \simeq (\sigma_0^2\mathbf{a}_0\mathbf{a}_0^{\mathrm{H}})\bar{\mathbf{a}} \simeq \sigma_0^2(\mathbf{a}_0^{\mathrm{H}}\bar{\mathbf{a}})\mathbf{a}_0, \tag{8}$$

is proportional to the desired signal's SV (note that the nominal SV is usually a good enough approximation so that $\mathbf{a}_0^H\bar{\mathbf{a}}$ is far from zero).

3.3. NPIC Matrix Reconstruction Using CG

The classical least mean square (LMS) algorithm in [20] is based on adjusting the array of sensors in real-time toward a signal coming from the desired direction while the interferences are attenuated. The LMS algorithm is an CG algorithm which searches for the minimum of a quadratic cost function. We apply LMS to solve the MVDR optimization problem in (3) by using the Lagrange multiplier α to include the constraint into the objective function as

$$J(\mathbf{w}) = \mathbf{w}^{\mathrm{H}}\hat{\mathbf{R}}_{\mathrm{i+n}}\,\mathbf{w} + \alpha(\mathbf{w}^{\mathrm{H}}\hat{\mathbf{a}}_0 - 1). \tag{9}$$

The cost function $J(\mathbf{w})$ can be minimized by applying the steepest descent algorithm as follows

$$\mathbf{w}(t+1) = \mathbf{w}(t) - (1/2)\mu\nabla J(\mathbf{w}), \tag{10}$$

where $\nabla J(\mathbf{w})$ is the gradient of the cost function with respect to $\mathbf{w}(t)$. The gradient vector can be obtained from (9) as

$$\nabla J(\mathbf{w}) = 2\hat{\mathbf{R}}_{\mathrm{i+n}}\mathbf{w}(t) + \alpha\hat{\mathbf{a}}_0. \tag{11}$$

Exploiting (5) over angular $\bar{\Theta}$ and the MEPS estimate (6), the NPIC matrix can be reconstructed by numerically evaluating

$$\hat{\mathbf{R}}_{\mathrm{i+n}} = \int_{\bar{\Theta}} \hat{P}_{\mathrm{meps}}\mathbf{a}(\theta)\mathbf{a}^{\mathrm{H}}(\theta)d\theta, \tag{12}$$

Sampling $\bar{\Theta}$ uniformly with Q sampling points spaced by $\Delta\theta$, (12) can be approximated by

$$\hat{\mathbf{R}}_{\mathrm{i+n}} \approx \sum_{i=1}^{Q} \frac{\mathbf{a}(\theta_i)\mathbf{a}^{\mathrm{H}}(\theta_i)}{\epsilon_p|\mathbf{a}^{\mathrm{H}}(\theta)\hat{\mathbf{R}}^{-1}\mathbf{u}_1|^2}\Delta\theta. \tag{13}$$

Here and in the next section we show how to apply the update (10) while avoiding to compute (13) explicitly. Rewriting (11) by substituting the expression for $\hat{\mathbf{R}}_{\mathrm{i+n}}$, we get

$$\nabla J(\mathbf{w}) = \sum_{i=1}^{Q} 2\hat{P}_{\mathrm{meps}}\Big(\mathbf{a}^{\mathrm{H}}(\theta_i)\mathbf{w}(t)\Big)\mathbf{a}(\theta_i)\Delta\theta + \alpha\hat{\mathbf{a}}_0, \tag{14}$$

By substituting (14) into (10) and rearranging, we obtain a recursion for the beamformer given by

$$\mathbf{w}(t+1) = \mathbf{w}(t) - \mu\big[\hat{\mathbf{a}}_0 - \mathbf{r}(t)\big], \tag{15}$$

where $\mathbf{r}(t) = \sum_{i=1}^{Q} \hat{P}_{\mathrm{meps}}\Big(\mathbf{a}^{\mathrm{H}}(\theta_i)\mathbf{w}(t)\Big)\mathbf{a}(\theta_i)\Delta\theta$ and μ is the steepest descent step size. To find the beamformer, the conjugate gradient algorithm is used to solve the unconstrained quadratic programming problem in (9) as algorithm 1.

Algorithm 1 Conjugate Gradient [21]

1: Choose an initial iterate $\mathbf{w}_0$;
2: Set $\mathbf{g}_0 = \nabla J(\mathbf{w}_0)$ and $\mathbf{e}_0 = -\mathbf{g}_0$;
3: Set $t \leftarrow 0$;
4: **while** $\|\nabla J(\mathbf{w}_t)\| > \mathrm{tol.}$ **do**
5: Define $\mathbf{e}_t = \hat{\mathbf{a}}_0 - \mathbf{r}_t$
6: Determine the step-size $\mu_t = -\dfrac{\mathbf{e}_t^{\mathrm{T}}\mathbf{g}_t}{\mathbf{e}_t^{\mathrm{T}}\hat{\mathbf{R}}_{\mathrm{i+n}}\mathbf{e}_t}$
7: $\mathbf{w}_{t+1} = \mathbf{w}_t + \mu_t\mathbf{e}_t$;
8: $\mathbf{g}_{t+1} = \nabla J(\mathbf{w}_{t+1})$
9: Determine β_t as $\beta_t = \dfrac{\mathbf{g}_{t+1}^{\mathrm{T}}(\mathbf{g}_{t+1} - \mathbf{g}_t)}{\mathbf{g}_t^{\mathrm{T}}\mathbf{g}_t}$;
10: Set $\mathbf{e}_{t+1} = -\mathbf{g}_{t+1} + \beta_t\mathbf{e}_t$;
11: Set $t \leftarrow t+1$;
12: **end while**

Hence, the weight vector is updated at each iteration by the recursion in (15) for reducing the complexity.

Up to now, the main difference here from prior works lies in the fact that the integral (12) is approximated by a summation (13), which would require a complexity of $\mathcal{O}(M^2Q)$ to be able to synthesize narrowband signal's power accurately. However, in the computation of (14) and in the final proposed algorithm in (15), (19) and (22) (see below), we avoid actually computing expensive $O(M^2)$ outer products, so our algorithm requires $\mathcal{O}(MQ)$ for steps (2,4,5,7,8 and 10) while steps in (6 and 9) needs $\mathcal{O}(M)$ complexity. Since this algorithm iterates t times to finding the best step-size, μ_t. Hence, the final computational complexity of the proposed method is only $\mathcal{O}(tMQ)$ while computing the beamformer without need for the inverse of the NPIC matrix.

3.4. MEPS Estimation Using CG

In order to compute (13) efficiently, we can use an iterative solution to the linear system, and take advantage of the structure of the SCM, $\hat{\mathbf{R}}$. We write the term $\mathbf{v} = \hat{\mathbf{R}}^{-1}\mathbf{u}_1$ in (13) and consider the optimization problem

$$\min_{\mathbf{v}} \mathbf{v}^{\mathrm{H}}\hat{\mathbf{R}}\,\mathbf{v} \quad \text{s.t.} \quad \mathbf{v}^{\mathrm{H}}\mathbf{u}_1 = 1, \tag{16}$$

The corresponding CG algorithm is described by

$$\mathbf{v}(t+1) = \mathbf{v}(t) + \xi(\mathbf{u}_1 - \hat{\mathbf{R}}\mathbf{v}(t)), \tag{17}$$

where ξ is a step size. Now, substituting the expression for $\hat{\mathbf{R}}$ and multiplying by $\mathbf{v}(t)$ yields

$$\hat{\mathbf{R}}\mathbf{v}(t) = \frac{1}{K}\sum_{t=1}^{K}\mathbf{x}(t)\Big(\mathbf{x}^{\mathrm{H}}(t)\mathbf{v}(t)\Big), \tag{18}$$

By substituting (18) into (17), we obtain

$$\mathbf{v}(t+1) = \mathbf{v}(t) + \xi\Big(\mathbf{u}_1 - \sum_{t=1}^{K}\mathbf{x}(t)\big(\frac{\mathbf{x}^{\mathrm{H}}(t)\mathbf{v}(t)}{K}\big)\Big). \tag{19}$$

In (17), the step size, ξ, should satisfy $0 < \xi < 2/\lambda_{\max}$ ($\lambda_{\max}$ is the largest eigenvalue of $\hat{\mathbf{R}}$), with fastest convergence occurring for $\xi \approx 1/\lambda_{\max}$. Since computing $\lambda_{\max}$ requires $\mathcal{O}(M^3)$ operations, it is more efficient to use an approximation. Assume that λ is an eigenvalue of $\hat{\mathbf{R}}$ with respect to the eigenvalue $\mathbf{z}$, so we can write

$$\lambda\mathbf{z} = \hat{\mathbf{R}}\mathbf{z} = \frac{1}{K}\sum_{t=1}^{K}\mathbf{x}(t)\Big(\mathbf{x}^{\mathrm{H}}(t)\mathbf{z}\Big). \tag{20}$$

Taking norm in both sides

$$\begin{aligned} \mid \lambda \mid\parallel \mathbf{z} \parallel &= \frac{1}{K} \parallel \sum_{t=1}^{K}\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)\mathbf{z} \parallel \\ &\leqslant \frac{1}{K}\sum_{t=1}^{K} \parallel \mathbf{x}(t) \parallel\mid \mathbf{x}^{\mathrm{H}}(t)\mathbf{z} \mid \leqslant \frac{1}{K}\sum_{t=1}^{K} \parallel \mathbf{x}(t) \parallel^2\parallel \mathbf{z} \parallel, \end{aligned} \tag{21}$$

Hence $\mid \lambda \mid \leqslant (1/K)\sum_{t=1}^{K} \parallel \mathbf{x}(t) \parallel^2$. An approximation to the step size, ξ, is given by

$$\xi \approx \frac{K}{\sum_{t=1}^{K} \parallel \mathbf{x}(t) \parallel^2}. \tag{22}$$

The computational complexity of the proposed MEPS-NPIC-CG is $\mathcal{O}(tMQ)$. The solution of the quadratically constrained quadratic programming (QCQP) problem in [11] has complexity of at least $\mathcal{O}(M^{3.5} + M^2Q)$, while the beamformer in [8] has a complexity of $\mathcal{O}(KM) + \mathcal{O}(M^3)$ and the reconstructed NPIC matrices in [13] and [16] have a complexity of $\mathcal{O}(M^3)$. Also, the cost of the beamformer in [22] is $\mathcal{O}(\max(M^2Q, M^{3.5}))$.

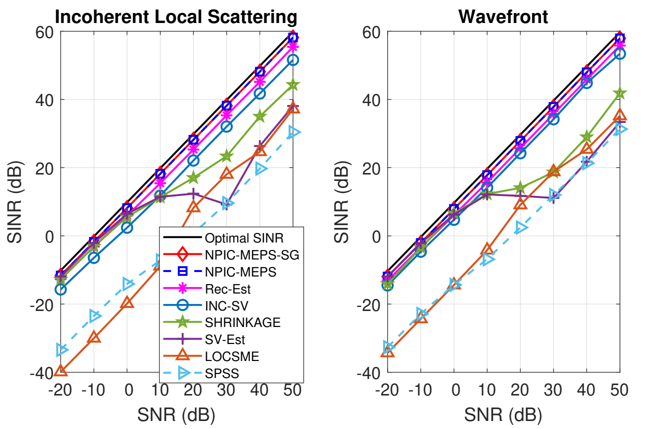

Fig. 1. SINR vs SNR a) Incoherent Local Scattering b) Wavefront Distortion

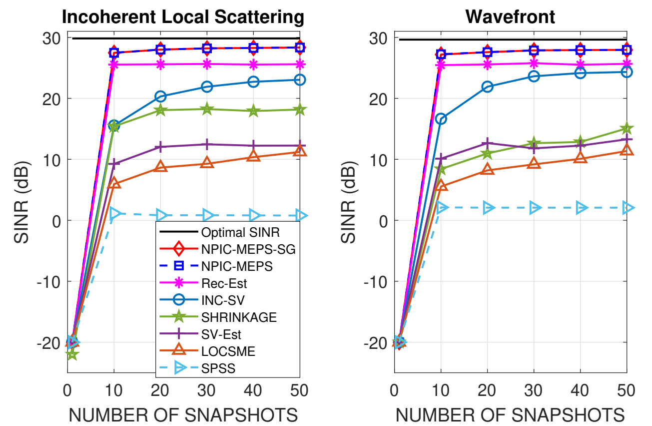

Fig. 2. SINR vs Snapshots a) Incoherent Local Scattering b) Wavefront Distortion

4. SIMULATIONS

In this section, a uniform linear array of $M = 10$ omnidirectional sensors and half-wavelength interelement spacing is considered. Two interferers and a DS impinge on the sensor array with incident angles 50^o, 20^o and 5^o, respectively. The interference-plus-noise ratio (INR) for each interferer is assumed 30 dB in each sensor. The additive noise is modeled as spatially white Gaussian, where 100 Monte Carlo runs are performed for each simulation. When we examine the performance of the output SINR versus input SNR, the number of snapshots is set to $K = 30$ whereas for the performance comparison of the adaptive beamformers versus the number of snapshots the SNR is set to 20 dB. The proposed MEPS-NPIC-CG method is compared with LOCSME [13], the modified projection beamformer (Shrinkage) [8], the reconstruction-estimation based beamformer (Rec-Est) [11], the SPSS beamformer in [16], the algorithm based on noise-plus-interference covariance matrix reconstruction and SV estimation (INC-SV), [22] and the beamformer (SV-Est) in [23]. The angular sector of the DS is set to $\Theta = [-1^\circ, 11^\circ]$ while the interference angular sector is $\bar{\Theta} = [-90^\circ, -1^\circ) \cup (11^\circ, 90^\circ]$. For the proposed MEPS-NPIC-CG beamformer, $\mathrm{tol} = 0.001$, $t = 7$, $S = 10$ and $Q = 90$ are used and the bound for the beamformer in [22] is set as $\epsilon = \sqrt{0.1}$. To solve all convex optimization problems the Matlab CVX toolbox [24] is used.

In the first scenario, the desired signal SV is distorted by incoherent local scattering effects so that the actual SV is assumed to have a time-varying signature and the SV is expressed as $\mathbf{a}(t) = s_0(t)\mathbf{a}(\theta_0) + \sum_{p=1}^{4} s_p(t)\mathbf{a}(\theta_p)$ where $s_0(t)$ and $s_p(t)$ $(p = 1, 2, 3, 4)$ are independently and identically distributed (i.i.d.) zero-mean complex Gaussian random variables independently drawn from a random generator. The angles θ_p $(p = 0, 1, 2, 3, 4)$ are drawn independently in each simulation run from a uniform generator with mean 5° and standard deviation 2°. Note that θ_p changes from run to run while remains fixed from snapshot to snapshot. At the same time, the random variables $s_0(t)$ and $s_p(t)$ change not only from run to run but also from snapshot to snapshot. Figs. 1(a) and Figs. 2(a) depict the output SINR of the tested beamformers versus the SNR and snapshots under the incoherent local scattering case. It is demonstrated that the MEPS-NPIC-SG has high accuracy SINR for all snapshots. Also, it is seen that the performance of the proposed (MEPS-NPIC-SG) method is same as our method given in [18] while outperforms the other beamformers.

In the second scenario, we consider the situation when the signal SV is distorted by wave propagation effects in an inhomogeneous medium. Specifically, independent-increment phase distortions are accumulated by the components of the presumed SV. It is assumed that the phase increments remain fixed in each simulation run and are independently chosen from a Gaussian random generator with zero mean and standard deviation 0.07. Figs. 1(b) and Figs. 2(b) show the output SINR of the beamformers versus the input SNR for snapshot. Similar to the previous scenario, the proposed beamformer significantly outperforms other beamformers due to its ability to reconstruct the NPIC matrix and estimate the desired signal SV with higher accuracy than other methods.

5. CONCLUSION

A low-complexity approach to robust adaptive beamforming based on estimated weight vector through CG recursions, named MEPS-NPIC-CG, has been proposed. The computed weight vector is exploited to reconstruct accurate NPIC matrix without requiring matrix inversions. Simulations demonstrate that MEPS-NPIC-CG can offer a superior performance to recently reported robust adaptive beamforming methods.

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