COMPARATOR NETWORK AIDED DETECTION FOR MIMO RECEIVERS WITH 1-BIT QUANTIZATION

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ABSTRACT

In this study, we propose comparator network-aided MIMO systems with 1-bit ADCs at the receiver. During the uplink transmission, the received signals simultaneously flow into the 1-bit ADCs and the comparator network, where the latter is composed of several simple comparators and each comparator has binary output. Two comparator networks are proposed, namely, fully and partially connected networks. Based on the proposed system design, we develop a low-resolution aware linear minimum mean-squared error detector according to the Bussgang theorem. We also devise a greedy search-based partially connected network that can use much less comparators to approach the performance of the fully connected network. Simulation results show that by taking into account the additional comparator network the system can outperform the existing low-resolution detection scheme in terms of bit error rate.

***Index Terms*—** MIMO, 1-bit ADCs, comparator networks, Bussgang theorem, Greedy search

1. INTRODUCTION

As a promising technical candidate for next generation cellular systems, large-scale multiple-input multiple-output (MIMO) systems have attracted much attention due to its large improvement in spectral efficiency [1]. However, deploying such a large number of antennas at the base station (BS) will bring some practical challenges, such as hardware cost and power consumption. For example, the power consumption of analog-to-digital converters (ADCs) P_{ADC} scales exponentially in the number of quantization bits B, i.e., $P_{\text{ADC}} \in 2^B$ [2]. The use of current high-speed and high-resolution ADCs (8-12 bits) for each antenna array would become a great burden to the BS. Consequently, the use of antennas with low-resolution ADCs (1-3 bits) are promoted as a solution to this problem [3–5].

Specifically, 1-bit ADCs are of interest in large-scale MIMO systems since they demand very low power and obviate the use of automatic gain control (AGC) [6]. Several studies have investigated 1-bit ADCs in such systems. The works in [7–9] have proposed algorithms to estimate the channel statistics based on the coarsely quantized signal, such as the maximum a-posteriori probability (MAP), recursive least squares (RLS) and approximate message passing (AMP) algorithm. While performing the signal detection, iterative detection and decoding (IDD) [10] and sphere decoding [11] are proposed to estimate the transmitted symbols sent by the users. Furthermore, the works in [12–14] have applied temporal oversampling techniques into the system to achieve better estimation and detection performance. Recently, the authors in [15, 16] have used spatial oversampling by employing a one-bit $\Sigma\Delta$ sampling scheme. Simulation results have shown large performance gains on channel estimation and signal detection offered by the proposed approach. Unlike the spatial $\Sigma\Delta$ approach in [15, 16] which relies on spatial oversampling, the method in the present study is appropriate for all MIMO channels with coarse quantization at the receiver. Moreover, the proposed method does not imply feedback loops and is compatible with the established Bussgang decomposition approach.

Different from prior works on 1-bit MIMO systems, in this work we propose a comparator network based system design, where the received signals simultaneously flow into the comparator network and the 1-bit ADCs. Both of them have binary outputs. The comparator network consists of simple comparator circuits which compare different signals from different antennas elements. The comparator network can be interpreted as an extension of the channel by linear combinations of its outputs with a subsequent quantization step. Different to the case where the antenna output signals are sampled with full resolution in amplitude these linear combinations can contain useful information which can aid the detection process. For the comparator network aided receiver, we develop a low-resolution aware (LRA) linear minimum mean squared error (LMMSE) detector based on the Bussgang theorem. Two types of comparator networks are considered, fully and partially connected, where the latter is based on a greedy search. Numerical results confirm that the BER can be significantly reduced when utilizing a comparator network.

The rest of this paper is organized as follows: Section 2 shows the system model and gives the insight of the comparator network. Section 3 and Section 4 derives the linear detector for the proposed system and illustrates the design algorithm for the comparator matrix $\mathbf{B}'$, respectively. In Section 5, the numerical results are presented and Section 6 concludes the paper.

Throughout the paper the following notations are used: the bold upper and lower case such as $\mathbf{A}$ and $\mathbf{a}$ denote matrices and vectors, respectively. $\mathbf{I}_n$ is a $n \times n$ identity matrix. Additionally, $\text{diag}(\mathbf{A})$ is a diagonal matrix only containing the diagonal elements of $\mathbf{A}$. The vector or matrix transpose and conjugate transpose are represented by $(\cdot)^T$ and $(\cdot)^H$. $\sin^{-1}(\cdot)$ denotes the inverse of sine function.

2. SYSTEM MODEL

The overall system model is illustrated with blocks in Fig. 1, where the received signal $\mathbf{y}$ for the uplink single-cell MIMO system with N_t single-antenna users and N_r receive antennas is written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}. \tag{1}$$

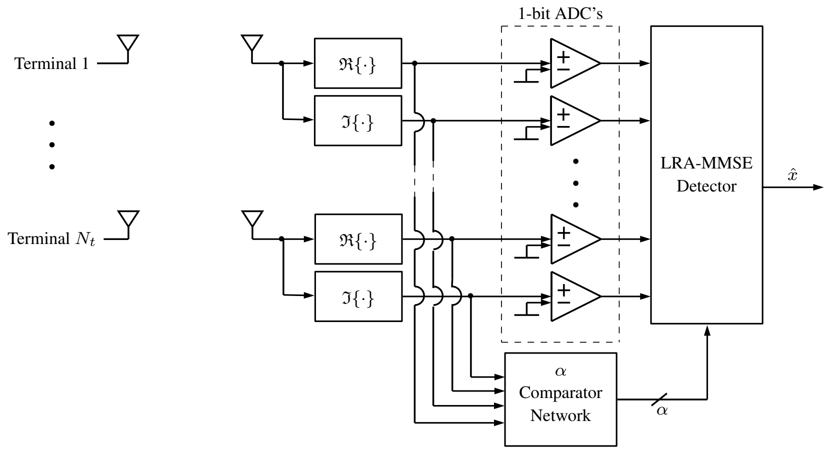

Fig. 1: System model of multi-user MIMO with 1-bit ADCs and an additional comparator network

The vector $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$ contains identically independent distributed (IID) transmit symbols, each of which has unit power. $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix and $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is the noise vector. Using the transformation from a complex into a real-valued system, we obtain

$$\begin{bmatrix} \Re\{\mathbf{y}\} \\ \Im\{\mathbf{y}\} \end{bmatrix} = \begin{bmatrix} \Re\{\mathbf{H}\} & -\Im\{\mathbf{H}\} \\ \Im\{\mathbf{H}\} & \Re\{\mathbf{H}\} \end{bmatrix} \begin{bmatrix} \Re\{\mathbf{x}\} \\ \Im\{\mathbf{x}\} \end{bmatrix} + \begin{bmatrix} \Re\{\mathbf{n}\} \\ \Im\{\mathbf{n}\} \end{bmatrix}, \quad (2)$$

where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ are functions to get the real and imaginary part, respectively. The formula in (2) can be reformulated as

$$\mathbf{y}^R = \mathbf{H}^R \mathbf{x}^R + \mathbf{n}^R. \quad (3)$$

The received signal is then sent to the 1-bit ADCs and the comparator network (shown in Fig. 2). Each comparator compares two

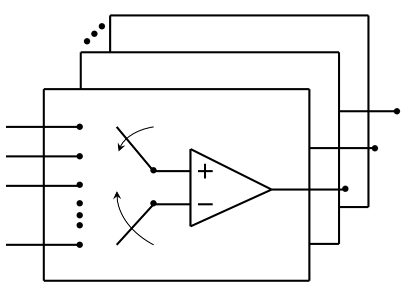

Fig. 2: Insight of the comparator network

received signals and quantizes the difference as $\{\pm 1\}$ based on a threshold. Letting $\mathcal{Q}(\cdot)$ represent the 1-bit quantization, the input of the detector is then

$$\mathbf{z}_{\mathcal{Q}}^R = \mathcal{Q}\left(\begin{bmatrix} \mathbf{y}^R \\ \mathbf{B}'\mathbf{y}^R \end{bmatrix} \right) = \mathcal{Q}\left(\begin{bmatrix} \mathbf{I}_{2N_r} \\ \mathbf{B}' \end{bmatrix} \mathbf{y}^R \right), \quad (4)$$

where $\mathbf{B}' \in \mathbb{R}^{\alpha \times 2N_r}$ represents the comparator network and has the form as

$$\mathbf{B}' = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & -1 & 0 & \cdots & 0 \\ 1 & 0 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 \end{bmatrix}. \quad (5)$$

In each row of $\mathbf{B}'$, there is only one pair of 1 and -1 and the remaining entries are zeros. With $\mathbf{B} = \left[\mathbf{I}_{2N_r}; \mathbf{B}'\right]$, (4) reads as

$$\mathbf{z}_{\mathcal{Q}}^R = \mathcal{Q}\left(\mathbf{B}\mathbf{y}^R\right). \quad (6)$$

The novelty of the present study is that 1-bit samples and the comparator output signals, described by $\mathcal{Q}(\mathbf{B}'\mathbf{y}^R)$, are jointly used for the detection process.

3. LINEAR DETECTION

Based on the proposed system model in (6), the corresponding linear receiver is derived to reconstruct the transmitted symbols. The optimization problem for getting the optimal linear receiver is formulated as

$$\mathbf{W}_{\text{LRA-MMSE}} = \arg\min_{\mathbf{W}} \ E\left[\left\| \mathbf{x}^R - \mathbf{W}^H \mathbf{z}_{\mathcal{Q}}^R \right\|_2^2 \right], \quad (7)$$

where $\mathbf{W} \in \mathbb{R}^{(2N_r+\alpha) \times 2N_t}$. The solution is

$$\mathbf{W}_{\text{LRA-MMSE}} = \mathbf{C}_{\mathbf{z}_{\mathcal{Q}}^R}^{-1} \mathbf{C}_{\mathbf{z}_{\mathcal{Q}}^R \mathbf{x}^R}, \quad (8)$$

where the involved covariance matrix $\mathbf{C}_{\mathbf{z}_{\mathcal{Q}}^R}$ is calculated as [17]

$$\mathbf{C}_{\mathbf{z}_{\mathcal{Q}}^R} = \frac{2}{\pi} \sin^{-1}\left(\mathbf{K}\Re\{\mathbf{C}_{\mathbf{z}^R}\}\mathbf{K}\right), \text{with } \mathbf{K} = \text{diag}(\mathbf{C}_{\mathbf{z}^R})^{-\frac{1}{2}} \quad (9)$$

and the cross-correlation matrix $\mathbf{C}_{\mathbf{z}_{\mathcal{Q}}^R \mathbf{x}^R}$ is based on the Bussgang theorem [18]

$$\mathbf{C}_{\mathbf{z}_{\mathcal{Q}}^R \mathbf{x}^R} = \sqrt{\frac{2}{\pi}} \mathbf{K} \mathbf{C}_{\mathbf{z}^R \mathbf{x}^R} = \sqrt{\frac{2}{\pi}} \frac{1}{2} \mathbf{K}\mathbf{B}\mathbf{H}^R. \quad (10)$$

The auto-correlation of $\mathbf{z}^R$ is

$$\mathbf{C}_{\mathbf{z}^R} = \frac{1}{2} \mathbf{B}\mathbf{H}^R {\mathbf{H}^R}^H \mathbf{B}^T + \frac{\sigma_n^2}{2} \mathbf{B}\mathbf{B}^T. \quad (11)$$

4. DESIGN OF COMPARATOR NETWORK

In this section, the matrix design of the comparator network in (5) is illustrated. Two types of networks are considered, namely, fully and partially connected networks.

4.1. Fully Connected Network

In this network, every two of the received signals are compared, so that there are overall $\alpha_f = C^2_{2N_r} = N_r(2N_r-1)$ comparators. For instance, if a system is with $N_r = 2$ receive antennas, $\alpha_f = C^2_4 = 6$ comparators are needed in this network and the corresponding matrix $\mathbf{B}'$ is described by

$$\mathbf{B}' = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}. \tag{12}$$

4.2. Partially Connected Network

The main drawback of the fully connected network is the massive use of the comparators., where the number of comparators α_f is proportional to the square of the number of receive antennas N_r. For large-scale MIMO system, it will need much more comparators in the fully connected network. In order to increase the usage efficiency of the comparators, the partially connected network is proposed, where the number of utilized comparators α_p is only a small fraction of what is required for the fully connected network.

In this subsection, two types of network design are considered, random and greedy search based. The former is to randomly select α_p out of α_f ($\alpha_p << \alpha_f$) comparators, while the selection criterion of the latter is the mean square error (MSE). The proposed greedy search algorithm implies that in each search cycle, the comparator configuration with the highest MSE reduction is selected. The detailed process is summarized in Algorithm 1.

Algorithm 1 MMSE based Greedy Search

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1:  Find the fully connected network B′ in (12) and get the number of rows,
    defined by R_max
2:  Extract the first α_p rows of B′, defined by B′_{α_p}
3:  Constitute B in (6) and calculate W in (8)
4:  Compute the MSE with E[||x^R − W^H z^R_Q||_2^2], defined by l_min
5:  for i = 1 : α_p do
6:      Take the ith row of B′_{α_p} and freeze the other α_p − 1 rows
7:      for j = 1 : R_max do
8:          if the jth row of B′ is already in B′_{α_p} then
9:              j = j + 1
10:         else
11:             Replace the ith row of B′_{α_p} with the jth row of B′
12:             Constitute B and calculate W
13:             Compute the MSE value, defined by l
14:             if l < l_min then
15:                 l_min = l
16:                 Update B′_{α_p}
17:             end if
18:         end if
19:     end for
20: end for
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5. NUMERICAL RESULTS

In this section, an uplink single-cell 1-bit MIMO system with $N_t = 2$ is considered. The modulation scheme is Quadrature Phase-Shift Keying (QPSK) and the SNR is defined as $10\log(\frac{1}{\sigma_n^2})$. The BER performance plots are obtained by taking the average of 2000 different channel matrices, noise and symbol vectors. While making the signal detection, the LRA-MMSE detector is applied in the system.

The BER performance of fully and partially connected networks under perfect channel state information (CSI) are shown in Fig. 3, where the system with fully connected network achieves the best BER performance with the cost of a large number of comparators. In the partially connected networks, the MMSE based greedy search outperforms the random selection approach especially at high SNR, where the error floor is eliminated. A surprising observation is that the greedy search approach has almost the same BER performance as the fully connected method but with much less comparators. This shows great advantages of the greedy search based partially connected network. However, also the approach with the comparator network using random selected inputs is beneficial in terms of BER. While making comparison with the approach without additional comparator network, it can be seen that by adding extra 20 comparators the performance gain is significant and the error floor goes down largely.

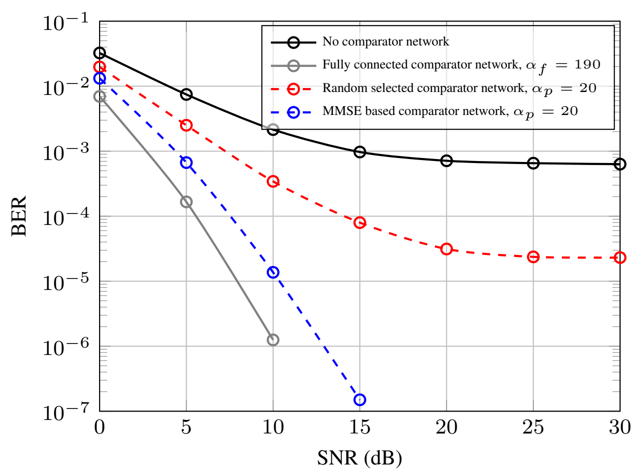

Fig. 3: BER performance in 2×10 MIMO systems.

However, it should be mentioned that although the greedy search approach yields comparable good BER performance and less usage of the comparators, the computational cost is the highest among the approaches listed in Fig. 3 due to its iterative search for the least MSE values. The computational complexity of different approaches is summarized in Table 1, where $\mathcal{O}(\cdot)$ is the big O notation. As the example showed in Fig. 3, the required computational (arithmetic operations) and hardware costs (additional comparators) are approximately calculated in Table 2.

6. CONCLUSIONS

In this study, comparator network based 1-bit MIMO systems are proposed. The additional comparator network provides additional information about the received signal which can be used to reduce the BER performance with only a slight increase in hardware cost and required computational complexity. Two types of comparator networks are proposed, fully and partially connected networks. Simulation results show that the proposed partially connected networks, especially the MMSE based greedy search approach, require less comparators while introducing small performance degradation compared with the proposed fully connected networks. As a future research topic it would be interesting to develop a search algorithm for

Table 1: Computational complexity

Approach	Network Design	LRA-MMSE Detection
No network	-	$\mathcal{O}((2N_r)^3 + 2N_t(2N_r)^2 + 4N_rN_t)$
Full connection	-	$\mathcal{O}((2N_r+\alpha_f)^3 + 2N_t(2N_r+\alpha_f)^2 + 2N_t(2N_r+\alpha_f))$
MMSE based Greedy search	$\mathcal{O}(\alpha_p(R_{\max}-\alpha_p)(2N_r+\alpha_p)^3)$	$\mathcal{O}((2N_r+\alpha_p)^3 + 2N_t(2N_r+\alpha_p)^2 + 2N_t(2N_r+\alpha_p))$
Random selection	-	$\mathcal{O}((2N_r+\alpha_p)^3 + 2N_t(2N_r+\alpha_p)^2 + 2N_t(2N_r+\alpha_p))$

Table 2: Computational cost and hardware costs in terms of additional comparators in Fig. 3

Approach	Computational Cost	Hardware Cost
No network	$\mathcal{O}(9680)$	-
Full connection	$\mathcal{O}(9438240)$	190
MMSE based Greedy search	$\mathcal{O}(217670560)$	20
Random selection	$\mathcal{O}(70560)$	20

the design of the comparator network with low computational cost.

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