Iterative MMSE Space-Time Zero-Crossing Precoding for Channels With 1-Bit Quantization and Oversampling

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***Abstract*—Systems with 1-bit quantization and oversampling at the receiver are promising for IoT applications due to low hardware complexity and low energy consumption. Zero-crossing precoding implies that the information is conveyed in the time instance of a zero-crossing within the symbol time interval. In this context, this study proposes an iterative spatial temporal MMSE precoding algorithm. In comparison to the joint space time MMSE precoder, the proposed method shows a significantly lower computational complexity and a comparable MSE performance.**

I. Introduction

1-bit quantization and oversampling is promising for IoT applications [1], [2], where the most important requirements include low energy consumption and low hardware complexity.

In [3] it was shown that the achievable rate for bandlimited 1-bit quantized processes improves considerably with oversampling. In this context, a benefit of oversampling in terms of capacity per unit cost is shown in [4] and more recently, the achievable rate for the noisy bandlimited channel with oversampling was considered in [5].

A number of practical concepts for the 1-bit oversampling channel have been developed for SISO systems [6], [7], [8], [9], [10] and MIMO systems [11], [12]. A promising waveform design for MIMO systems is based on the zero-crossing precoding concept [12]. In this context, the MMSE criterion is beneficial in the low-SNR regime as shown in [13]. However, the joint space time precoding method in [13] involves the inversion of a large matrix. Iterative precoding methods have been considered in the literature to reduce the computational complexity of linear filters such as the MMSE and the zero forcing (ZF) filters. In [14], the inverse Gram matrix in the spatial MMSE filter is approximated by means of the diagonal band Newton iteration method. The coordinate descent method is applied in [15] to avoid the inversion of a high-dimensional Gram matrix in the design of decentralized massive MIMO precoders. Numerical results confirm that the mentioned iterative approaches have a comparable performance to the corresponding closed-form solutions.

Unlike existing iterative methods, the present study proposes an alternating approach involving two separate precoding matrices for space and time. In this context, a joint optimization problem is formulated and solved iteratively with a gradient projection method.

The rest of the paper is organized as follows: The system model is introduced in Section II. Then, Section III details the proposed space-time MMSE precoding. The simulation results and the conclusion are provided in Section IV.

Notation: In the paper all scalar values, vectors and matrices are represented by: a, $\boldsymbol{x}$ and $\boldsymbol{X}$, respectively.

II. System Model

A multiuser MIMO downlink channel as presented in Fig. 1 is considered. It is considered that the base station (BS) is equipped with N_{t} transmit antennas and that each of the N_{u} users is equipped with N_{r} receive antennas. Besides the signaling rate $\frac{M_{\mathrm{Tx}}}{T}$ and the sampling rate $\frac{M_{\mathrm{Rx}}}{T} = \frac{M\, M_{\mathrm{Tx}}}{T}$ the system is characterized by transmit filters and receive filters with impulse responses $g_{\mathrm{Tx}}(t)$ and $g_{\mathrm{Rx}}(t)$, respectively. The combined waveform, determined by the transmit and receive filter, is given by the convolution of both filters $v(t) = g_{\mathrm{Tx}}(t) * g_{\mathrm{Rx}}(t)$. Moreover, 1-bit quantization is applied in each receive chain. A frequency flat fading channel is considered which is described by a matrix $\boldsymbol{H}$ with dimensions $N_{\mathrm{r}}N_{\mathrm{u}} \times N_{\mathrm{t}}$.

It is considered that each user receives N_{r} independent data streams and the input associated to the ith antenna of the kth user is given by $\boldsymbol{x}_{k,i}$ with N complex symbols.

Due to the specific sampling and quantization properties of the receivers, each input sequence $\boldsymbol{x}_{k,i}$ is mapped to a desired binary output pattern, represented as a row vector $\boldsymbol{c}_{\mathrm{out},k,i}$ with a length of $N_{\mathrm{tot}} = M_{\mathrm{Rx}}N+1$ for further processing. Each symbol in the vector $\boldsymbol{x}_{k,i}$ is taken from an alphabet of $M_{\mathrm{Rx}}+1$ symbols. Depending on the symbol a zero-crossing is induced in one of the M_{Rx} oversampling time intervals or no zero-crossing is induced and the resulting sequence with all the desired zero-crossings is termed $\boldsymbol{c}_{\mathrm{out},k,i}$. Its construction is illustrated in [12], [13]. Stacking all $N_{\mathrm{r}}N_{\mathrm{u}}$ desired output patterns yields the matrix $\boldsymbol{C}_{\mathrm{out}}$ with dimensions $N_{\mathrm{r}}N_{\mathrm{u}} \times N_{\mathrm{tot}}$.

It is considered that the matrix $\boldsymbol{C}_{\mathrm{out}}$ serves as the input for a linear precoding scheme, where the spatial precoding is represented by matrix multiplication with $\boldsymbol{P}_{\mathrm{space}}$ from the left hand side and temporal precoding is represented by matrix multiplication with $\boldsymbol{P}_{\mathrm{time}}$ from the right hand side. Using these definitions, the unquantized received signals can be expressed

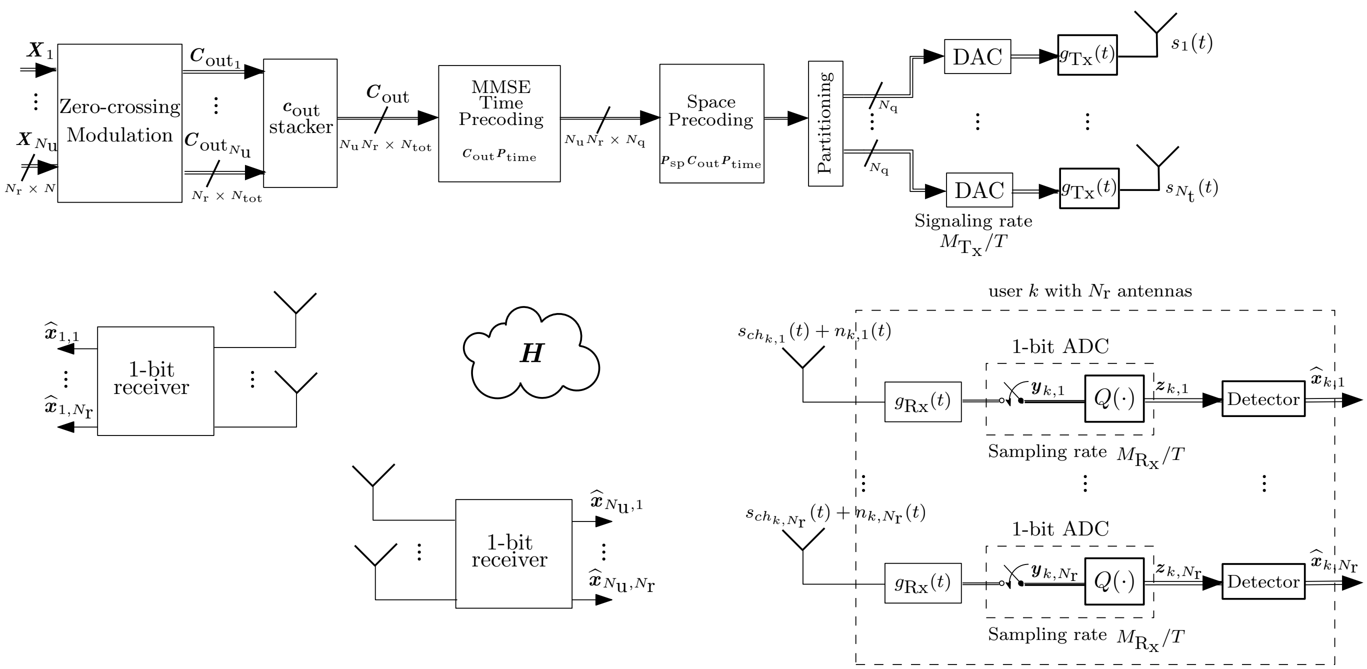

Fig. 1: Multiuser MIMO system model

with a stacked vector notation in terms of a matrix with dimensions $N_u N_r \times N_{tot}$ by

$$\begin{aligned} \boldsymbol{Y} =& \boldsymbol{H}\boldsymbol{P}_{space}\boldsymbol{C}_{out}\boldsymbol{P}_{time}\boldsymbol{U}^T\boldsymbol{V}^T + \boldsymbol{N}\boldsymbol{G}_{Rx}^T, \quad (1)\\ [\boldsymbol{Y}_1;\ldots;\boldsymbol{Y}_{N_u}] =& [\boldsymbol{H}_1;\ldots;\boldsymbol{H}_{N_u}]\boldsymbol{P}_{space}[\boldsymbol{C}_{out,1};\ldots;\boldsymbol{C}_{out,N_u}]\times\\ & \boldsymbol{P}_{time}\boldsymbol{U}^T\boldsymbol{V}^T + [\boldsymbol{N}_1;\ldots;\boldsymbol{N}_{N_u}]\boldsymbol{G}_{Rx}^T, \end{aligned}$$

where the temporal waveform is described by

$$\boldsymbol{V} = \begin{bmatrix} v(0) & v\left(\frac{T}{M_{Rx}}\right) & \cdots & v(TN) \\ v\left(-\frac{T}{M_{Rx}}\right) & v(0) & \cdots & v\left(T\left(N-\frac{1}{M_{Rx}}\right)\right) \\ \vdots & \vdots & \ddots & \vdots \\ v(-TN) & v\left(T\left(-N+\frac{1}{M_{Rx}}\right)\right) & \cdots & v(0) \end{bmatrix},$$

with dimensions $N_{tot} \times N_{tot}$. The receive filter with dimensions $N_{tot} \times 3N_{tot}$ is described by

$$\boldsymbol{G}_{Rx} = a_{Rx} \begin{bmatrix} [\ \boldsymbol{g}_{Rx}^T\ ] & 0\cdots & 0 \\ 0 & [\ \boldsymbol{g}_{Rx}^T\ ] & 0\cdots 0 \\ & \ddots\ \ddots\ \ddots & \\ 0\cdots & 0 & [\ \boldsymbol{g}_{Rx}^T\ ] \end{bmatrix}_{N_{tot}\times 3N_{tot}}, \quad (2)$$

with $\boldsymbol{g}_{Rx} = [g_{Rx}(-T(N+\frac{1}{M_{Rx}})), g_{Rx}(-T(N+\frac{1}{M_{Rx}})+\frac{T}{M_{Rx}}), \ldots, g_{Rx}(T(N+\frac{1}{M_{Rx}}))]^T$ and $a_{Rx} = (T/M_{Rx})^{1/2}$. The matrix $\boldsymbol{U}$ with dimensions $N_{tot} \times N_q$ is an upsampling matrix, which inserts zeros where $N_q = M_{Tx}N + 1$. It is defined by

$$\boldsymbol{U}_{m,n} = \begin{cases} 1, & \text{for} \quad m = M \cdot (n-1) + 1 \\ 0, & \text{else.} \end{cases} \quad (3)$$

The matrix $\boldsymbol{N}_k$ with dimensions $N_r \times 3N_{tot}$ contains i.i.d. complex Gaussian noise samples with zero mean and variance $\sigma_n^2 = N_0$. Finally, the reveived signal is quantized elementwise, using 1-bit resolution in the real part as in the imaginary part, which is expressed as

$$\boldsymbol{Z} = Q_1(\boldsymbol{Y}). \quad (4)$$

III. MMSE Space-Time Precoding

The aim is to find an *optimal* $\boldsymbol{P}_{space}$ with dimensions $N_t \times N_u N_r$ and an *optimal* $\boldsymbol{P}_{time}$ with dimensions $N_{tot} \times N_q$, that minimizes the MSE which is denoted as ϵ in the sequel. With the MMSE criterion and an instantaneous power constraint, the optimization problem can be cast as

$$\begin{aligned} \min_{f,\boldsymbol{P}_{time},\boldsymbol{P}_{space}} \quad & \mathrm{E}\{\|f\boldsymbol{Y} - \boldsymbol{C}_{out}\|_F^2\} \quad (5)\\ \text{subject to:} \quad & \|P_{space}C_{out}P_{time}U^T G_{Tx}\|_F^2 \leq E_0, \end{aligned}$$

where $\boldsymbol{G}_{Tx}$ represents the pulse shaping filter at the transmitter, which is constructed analogously to (2). The scalar E_0 denotes the maximum transmit energy for one transmission block and f is a scaling factor.

A. Iterative Space Time Precoding

The derivative of the objective function in (5) with respect to (w.r.t.) $\boldsymbol{P}_{space}^*$ is given by

$$\begin{aligned} \frac{\partial\epsilon}{\partial\boldsymbol{P}_{space}^*} =& f^2\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{P}_{space}\boldsymbol{C}_{out}\boldsymbol{P}_{time}\boldsymbol{U}^T\boldsymbol{V}^T\boldsymbol{V}\boldsymbol{U}\boldsymbol{P}_{time}^H\boldsymbol{C}_{out}^H\\ & - f\boldsymbol{H}^H\boldsymbol{C}_{out}\boldsymbol{V}\boldsymbol{U}\boldsymbol{P}_{time}^H\boldsymbol{C}_{out}^H. \quad (6) \end{aligned}$$

The derivative w.r.t. $\boldsymbol{P}_{time}^*$ is given by

$$\begin{aligned} \frac{\partial\epsilon}{\partial\boldsymbol{P}_{time}^*} =& f^2\boldsymbol{C}_{out}^H\boldsymbol{P}_{space}^H\boldsymbol{H}^H\boldsymbol{H}\boldsymbol{P}_{space}\boldsymbol{C}_{out}\boldsymbol{P}_{time}\boldsymbol{U}^T\boldsymbol{V}^T\boldsymbol{V}\boldsymbol{U}\\ & - f\boldsymbol{C}_{out}^H\boldsymbol{P}_{space}^H\boldsymbol{H}^H\boldsymbol{C}_{out}\boldsymbol{V}\boldsymbol{U}. \quad (7) \end{aligned}$$

Taking the derivative of ϵ w.r.t. f and equating it to zero yields

$$\begin{aligned} f =&(\mathrm{trace}\{\boldsymbol{H}\boldsymbol{P}_{\mathrm{space}}\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{V}^T\boldsymbol{C}_{\mathrm{out}}^H\} \\ &+ \mathrm{trace}\{\boldsymbol{C}_{\mathrm{out}}\boldsymbol{V}\boldsymbol{U}\boldsymbol{P}_{\mathrm{time}}^H\boldsymbol{C}_{\mathrm{out}}^H\boldsymbol{P}_{\mathrm{space}}^H\boldsymbol{H}^H\})/ \\ &(2(\|\boldsymbol{H}\boldsymbol{P}_{\mathrm{space}}\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{V}^T\|_F^2 \\ &+ \mathrm{trace}\{\boldsymbol{G}_{\mathrm{Rx}}\boldsymbol{R}_{\boldsymbol{N}}\boldsymbol{G}_{\mathrm{Rx}}^T\})). \end{aligned} \tag{8}$$

Based on the instantaneous power constraint in (5), it can be defined $Q = C_{\mathrm{out}}P_{\mathrm{time}}U^TG_{\mathrm{Tx}}$, $A = P_{\mathrm{space}}C_{\mathrm{out}}$, and $B = U^TG_{\mathrm{Tx}}$. With this, the power constraint can be further expressed as

$$\|P_{\mathrm{space}}Q\|_F^2 = \|AP_{\mathrm{time}}B\|_F^2 \leq E_0. \tag{9}$$

Hence, the spatial- and time-domain filters can be normalized to satisfy the instantaneous power constraint with equality as

$$\hat{P}_{\mathrm{space}} = P_{\mathrm{space}} \cdot \sqrt{E_0}\|P_{\mathrm{space}}Q\|_F^{-1} \tag{10}$$

$$\hat{P}_{\mathrm{time}} = P_{\mathrm{time}} \cdot \sqrt{E_0}\|AP_{\mathrm{time}}B\|_F^{-1}. \tag{11}$$

Using the introduced gradient expressions and normalizations from above a projected gradient descent algorithm is proposed as described in Algorithm 1.

Algorithm 1: Proposed Projected Gradient Descent

$i \leftarrow 0$
Initialize $P_{\mathrm{space}}[i] \leftarrow$ Spatial ZF
Initialize $P_{\mathrm{time}}[i]$, $f[i] \leftarrow$ Closed-form MMSE (23), (24)
repeat
  Update Precoding matrices:

  Caclulate $\boldsymbol{Q}[i] = C_{\mathrm{out}}P_{\mathrm{time}}[i]U^TG_{\mathrm{Tx}}$
  Calculate $\partial\epsilon/\partial\boldsymbol{P}_{\mathrm{space}}^*[i]$ by (6)
  $P_{\mathrm{space}}[i+1] \leftarrow P_{\mathrm{space}}[i] - \mu_{\mathrm{s}} \cdot \partial\epsilon/\partial\boldsymbol{P}_{\mathrm{space}}^*[i]$
  Normalize $P_{\mathrm{space}}[i+1]$ due to (10)

  Calculate $\boldsymbol{A}[i] = P_{\mathrm{space}}[i+1]C_{\mathrm{out}}$
  Calculate $\partial\epsilon/\partial\boldsymbol{P}_{\mathrm{time}}^*[i]$ by (7)
  $P_{\mathrm{time}}[i+1] \leftarrow P_{\mathrm{time}}[i] - \mu_{\mathrm{t}} \cdot \partial\epsilon/\partial\boldsymbol{P}_{\mathrm{time}}^*[i]$
  Normalize $P_{\mathrm{time}}[i+1]$ due to (11)

  Update f
  Calculate $f[i+1]$ by (8)

  $i \leftarrow i+1$
until convergence criterion triggers

B. Initialization with Spatial Zero-Forcing Precoding

For the initialization, we use a spatial zero-forcing precoder [16] with the precoding matrix given by $\boldsymbol{P}_{\mathrm{space}} = c_{\mathrm{ZF}}\boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{H}^H)^{-1}$, where c_{ZF} represents the prefactor. By considering a long block length, the matrix $\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{G}_{\mathrm{Tx}}\times \boldsymbol{G}_{\mathrm{Tx}}^T\boldsymbol{U}\boldsymbol{P}_{\mathrm{time}}^H\boldsymbol{C}_{\mathrm{out}}^H$ can be well approximated by a scaled identity matrix $a\boldsymbol{I}$. Imposing that the spatial precoding does not change the transmit power can be implicitly expressed as

$$a\ \mathrm{trace}\{\boldsymbol{P}_{\mathrm{space}}\boldsymbol{P}_{\mathrm{space}}^H\} = \mathrm{trace}\{a\ \boldsymbol{I}_{N_{\mathrm{r}}N_{\mathrm{u}}}\} = aN_{\mathrm{r}}N_{\mathrm{u}}, \tag{12}$$

which implies that the prefactor reads as

$$c_{\mathrm{ZF}} = \sqrt{N_{\mathrm{r}}N_{\mathrm{u}}/\mathrm{trace}\{(\boldsymbol{H}\boldsymbol{H}^H)^{-1}\}}. \tag{13}$$

With this scaling, the maximum transmit power can be expressed without the spatial precoding as

$$E_0 = \mathrm{trace}\{\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{G}_{\mathrm{Tx}}\boldsymbol{G}_{\mathrm{Tx}}^T\boldsymbol{U}\boldsymbol{P}_{\mathrm{time}}^H\boldsymbol{C}_{\mathrm{out}}^H\}. \tag{14}$$

Based on the ZF-precoding matrix the unquantized received signal is described by

$$\boldsymbol{Y} = c_{\mathrm{ZF}}\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{V}^T + \boldsymbol{N}\boldsymbol{G}_{\mathrm{Rx}}^T. \tag{15}$$

With this, the temporal MMSE problem can be cast as

$$\begin{aligned} \min_{f,\boldsymbol{P}_{\mathrm{time}}} \quad & \mathrm{E}\{\|f\boldsymbol{Y} - \boldsymbol{C}_{\mathrm{out}}\|_F^2\} \\ \text{subject to:} \quad & \|\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{G}_{\mathrm{Tx}}\|_F^2 \leq E_0, \end{aligned} \tag{16}$$

which is a similar problem as solved in [17]. By denoting the new MSE expression by ϵ_{T}, the Lagrangian function reads as

$$\begin{aligned} &L(\boldsymbol{P}_{\mathrm{time}}^*, \boldsymbol{P}_{\mathrm{time}}, f) = \\ &\epsilon_{\mathrm{T}} + \lambda(\mathrm{trace}\{\boldsymbol{C}_{\mathrm{out}}^H\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{G}_{\mathrm{Tx}}\boldsymbol{G}_{\mathrm{Tx}}^T\boldsymbol{U}\boldsymbol{P}_{\mathrm{time}}^H\} - E_0). \end{aligned} \tag{17}$$

The derivative w.r.t. $\boldsymbol{P}_{\mathrm{time}}^*$ yields

$$\begin{aligned} \frac{\partial L}{\partial \boldsymbol{P}_{\mathrm{time}}^*} =& f^2\ c_{\mathrm{ZF}}^2\ \boldsymbol{C}_{\mathrm{out}}^H\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{V}^T\boldsymbol{V}\boldsymbol{U} \\ &- f\ c_{\mathrm{ZF}}\ \boldsymbol{C}_{\mathrm{out}}^H\boldsymbol{C}_{\mathrm{out}}\boldsymbol{V}\boldsymbol{U} \\ &+ \lambda\boldsymbol{C}_{\mathrm{out}}^H\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{G}_{\mathrm{Tx}}\boldsymbol{G}_{\mathrm{Tx}}^T\boldsymbol{U}. \end{aligned} \tag{18}$$

Equating it to zero implies

$$\begin{aligned} \frac{c_{\mathrm{ZF}}}{f}\ \boldsymbol{C}_{\mathrm{out}}^H\boldsymbol{C}_{\mathrm{out}}\boldsymbol{V}\boldsymbol{U} =& c_{\mathrm{ZF}}^2\boldsymbol{C}_{\mathrm{out}}^H\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{V}^T\boldsymbol{V}\boldsymbol{U} \\ &+ \frac{\lambda}{f^2}\boldsymbol{C}_{\mathrm{out}}^H\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{G}_{\mathrm{Tx}}\boldsymbol{G}_{\mathrm{Tx}}^T\boldsymbol{U}, \end{aligned} \tag{19}$$

and

$$\frac{c_{\mathrm{ZF}}}{f}\ \boldsymbol{V}\boldsymbol{U} = c_{\mathrm{ZF}}^2\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{V}^T\boldsymbol{V}\boldsymbol{U} + \frac{\lambda}{f^2}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{G}_{\mathrm{Tx}}\boldsymbol{G}_{\mathrm{Tx}}^T\boldsymbol{U}.$$

The latter can be rearranged such that the structure of the optimal precoding matrix can be determined as

$$\boldsymbol{P}_{\mathrm{time}} = \frac{c_{\mathrm{ZF}}}{f}\ \boldsymbol{V}\boldsymbol{U}(\ c_{\mathrm{ZF}}^2\boldsymbol{U}^T\boldsymbol{V}^T\boldsymbol{V}\boldsymbol{U} + \frac{\lambda}{f^2}\boldsymbol{U}^T\boldsymbol{G}_{\mathrm{Tx}}\boldsymbol{G}_{\mathrm{Tx}}^T\boldsymbol{U})^{-1}.$$

The derivative w.r.t. f yields

$$\begin{aligned} \frac{\partial L}{\partial f} =& 2fc_{\mathrm{ZF}}^2\|\boldsymbol{V}\boldsymbol{U}\boldsymbol{P}_{\mathrm{time}}^H\boldsymbol{C}_{\mathrm{out}}^H\|_F^2 \\ &- c_{\mathrm{ZF}}\ 2\mathrm{Re}\{\mathrm{trace}\{\boldsymbol{C}_{\mathrm{out}}^H\boldsymbol{C}_{\mathrm{out}}\boldsymbol{V}\boldsymbol{U}\boldsymbol{P}_{\mathrm{time}}^H\}\} \\ &+ 2f\ \mathrm{trace}\{\boldsymbol{G}_{\mathrm{Rx}}\boldsymbol{R}_{\boldsymbol{N}}\boldsymbol{G}_{\mathrm{Rx}}^T\}. \end{aligned} \tag{20}$$

Equation it to zero yields

$$\begin{aligned} &\frac{c_{\mathrm{ZF}}}{f}\ \ \mathrm{trace}\{\boldsymbol{C}_{\mathrm{out}}^H\boldsymbol{C}_{\mathrm{out}}\boldsymbol{V}\boldsymbol{U}\boldsymbol{P}_{\mathrm{time}}^H\} = \\ &c_{\mathrm{ZF}}^2\|\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{V}^T\|_F^2 + \mathrm{trace}\{\boldsymbol{G}_{\mathrm{Rx}}\boldsymbol{R}_{\boldsymbol{N}}\boldsymbol{G}_{\mathrm{Rx}}^T\}, \end{aligned} \tag{21}$$

where the real part operator has been skipped due to the structure of the optimal $\boldsymbol{P}_{\mathrm{time}}$. After multiplication from the right with $\boldsymbol{P}_{\mathrm{time}}$ and applying the trace operator, (19) is equal to (21). Putting together the right hand sides yields

$$\frac{\lambda}{f^2} = \frac{\mathrm{trace}\{\boldsymbol{G}_{\mathrm{Rx}}\boldsymbol{R}_{\boldsymbol{N}}\boldsymbol{G}_{\mathrm{Rx}}^T\}}{\mathrm{trace}\{\boldsymbol{C}_{\mathrm{out}}^H\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^T\boldsymbol{G}_{\mathrm{Tx}}\boldsymbol{G}_{\mathrm{Tx}}^T\boldsymbol{U}\boldsymbol{P}_{\mathrm{time}}^H\}}, \tag{22}$$

where it is considered that the power constraint holds with equality such that $\mathrm{trace}\{\boldsymbol{C}_{\mathrm{out}}^{H}\boldsymbol{C}_{\mathrm{out}}\boldsymbol{P}_{\mathrm{time}}\boldsymbol{U}^{T}\boldsymbol{G}_{\mathrm{Tx}}\boldsymbol{G}_{\mathrm{Tx}}^{T}\boldsymbol{U}\boldsymbol{P}_{\mathrm{time}}^{H}\} = E_0$. With this, the temporal precoding matrix reads as

$$\boldsymbol{P}_{\mathrm{time}} = f^{-1}\ c_{\mathrm{ZF}}\ \boldsymbol{V}\boldsymbol{U}\boldsymbol{\Gamma}^{-1}, \tag{23}$$

with $\boldsymbol{\Gamma} = c_{\mathrm{ZF}}^{2}\boldsymbol{U}^{T}\boldsymbol{V}^{T}\boldsymbol{V}\boldsymbol{U} + \frac{\mathrm{trace}\{\boldsymbol{G}_{\mathrm{Rx}}\boldsymbol{R}_{\boldsymbol{N}}\boldsymbol{G}_{\mathrm{Rx}}^{T}\}}{E_0}\boldsymbol{U}^{T}\boldsymbol{G}_{\mathrm{Tx}}\boldsymbol{G}_{\mathrm{Tx}}^{T}\boldsymbol{U}$. Inserting (23) into the power constraint (14) yields

$$f = c_{\mathrm{ZF}}E_0^{-\frac{1}{2}}\ \|\boldsymbol{C}_{\mathrm{out}}\ \boldsymbol{V}\boldsymbol{U}\boldsymbol{\Gamma}^{-1}\boldsymbol{U}^{T}\boldsymbol{G}_{\mathrm{Tx}}\|_{F}. \tag{24}$$

IV. Numerical Results and Conclusion

For the numerical evaluation, a system with $N_{\mathrm{u}} = 2$ users with $N_{\mathrm{r}} = 2$ antennas and $N_{\mathrm{t}} = 5$ base station antennas is considered. Each transmit block consists of $N = 50$ symbols. The signaling and sampling rate are chosen as $M_{\mathrm{Rx}} = M_{\mathrm{Tx}} = 2$. As suggested in [11], the pulse shaping filter is a raised-cosine (RC) with roll-off factor $\epsilon_{\mathrm{Tx}} = 0.22$ and the receive filter is a root raised cosine (RRC) with roll-off factor $\epsilon_{\mathrm{Rx}} = 0.22$. The signal bandwidth is given by $W_{\mathrm{Tx}} = (1+\epsilon_{\mathrm{Tx}})/T$. Uncorrelated random channel coefficients are considered according to $h_{i,k,l} \sim \mathcal{CN}(0,1)$. The SNR is defined as

$$\mathrm{SNR} = \frac{E_0/(N_{\mathrm{q}}T)}{N_0\,(1+\epsilon_{\mathrm{Tx}})\,\frac{1}{T}} = \frac{E_0}{N_{\mathrm{q}}N_0\,(1+\epsilon_{\mathrm{Tx}})}. \tag{25}$$

The step size for the proposed projected gradient descent algorithm equals $\mu_{\mathrm{t}} = 10^{-7}$ and $\mu_{\mathrm{s}} = 10^{-3}$. In Fig. 2 the proposed iterative spatial temporal MMSE precoding algorithm is compared with the joint space time MMSE precoder [13] in terms of the MSE, which confirms that the MSE decreases with the iterations. The computational complexity of the precoder in [13] is on the order of $\mathcal{O}((N_{\mathrm{t}}N_{\mathrm{q}})^3)$. With $\mathcal{O}((N_{\mathrm{r}}N_{\mathrm{u}})^3 + N_{\mathrm{q}}^3 + i_{\max}(2N_{\mathrm{tot}}N_{\mathrm{u}}N_{\mathrm{r}}N_{\mathrm{q}}) + N_{\mathrm{tot}}(N_{\mathrm{u}}N_{\mathrm{r}})^2 + N_{\mathrm{tot}}N_{\mathrm{t}}N_{\mathrm{r}}N_{\mathrm{u}} + N_{\mathrm{t}}^2N_{\mathrm{r}}N_{\mathrm{u}} + (N_{\mathrm{r}}N_{\mathrm{u}})^2N_{\mathrm{t}} + N_{\mathrm{tot}}N_{\mathrm{t}}N_{\mathrm{r}}N_{\mathrm{u}} + N_{\mathrm{tot}}N_{\mathrm{t}}N_{\mathrm{q}} + N_{\mathrm{t}}^2N_{\mathrm{r}}N_{\mathrm{u}} + N_{\mathrm{tot}}N_{\mathrm{t}}N_{\mathrm{r}}N_{\mathrm{u}} + N_{\mathrm{q}}^2N_{\mathrm{r}}N_{\mathrm{u}} + N_{\mathrm{tot}}N_{\mathrm{q}}N_{\mathrm{r}}N_{\mathrm{u}})$, which is approximately $\mathcal{O}(N_{\mathrm{q}}^3)$ for small $i_{\max}$, the proposed method has a significantly lower computational complexity, in comparison to the method in [13]. Fig. 3 illustrates the uncoded bit error rate. The different precoding methods are compared in combination with the zero-crossing approach and the forward mapping approach presented in [11]. The results show a significant benefit for the zero-crossing approach. Finally, it is shown that the proposed iterative approach has a comparable bit error rate as compared to the closed form approach in [13] while having a significantly lower computational complexity.

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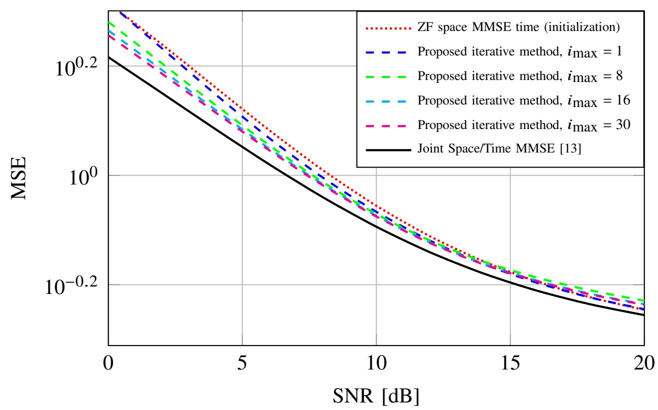

Fig. 2: MSE cost function for different precoding strategies

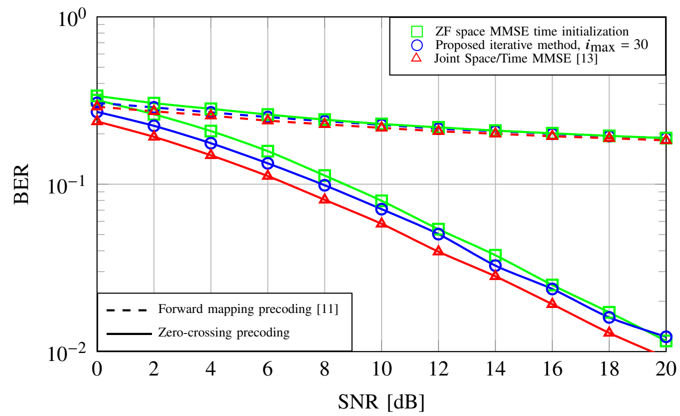

Fig. 3: BER performance

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