

DYNAMIC OVERSAMPLING IN 1-BIT QUANTIZED ASYNCHRONOUS LARGE-SCALE MULTIPLE-ANTENNA SYSTEMS FOR SUSTAINABLE IOT NETWORKS

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ABSTRACT

In this paper, we propose a dynamic oversampling technique for asynchronous large-scale multiple-antenna systems with 1-bit analog-to-digital converters at the base station that is suitable for sustainable internet of things and cellular networks. To the best of our knowledge, this is the first paper to introduce a dynamic oversampling technique for such systems. The main idea is to sample the received signal at a higher rate and only few weighted samples are chosen for further signal processing. We apply the generalized eigenvalue decomposition algorithm for linearly combining the samples and performing dimension reduction. We investigate the proposed technique in terms of the Bussgang theorem based sum rate capacity. Numerical results show that with the proposed dynamic oversampling technique the system can use small number of processing samples to achieve the same sum rates as the standard uniform oversampling technique while maintaining the same power consumption.

Index Terms— Asynchronous large-scale MIMO, 1-bit ADCs, dynamic oversampling, sum rate capacity

1. INTRODUCTION

The use of low-resolution analog-to-digital converters (ADCs) in large-scale multiple-antenna or multiple-input multiple-output (MIMO) systems, results in substantial reduction in hardware complexity and energy consumption at the base station (BS) [1, 2]. In particular, the case of extreme 1-bit ADCs, which consist of a simple analog comparator and consume only a few milliwatts, is appealing since they do not require automatic gain control (AGC). Hence the corresponding radio frequency (RF) chains can be implemented with very low cost and power consumption, which is important for large-scale MIMO systems and future sustainable networks.

Several recent works have studied large-scale MIMO systems with 1-bit ADCs at the front-end. Channel estimation is a challenging problem, since the magnitude and phase information about the received signal are severely distorted due to the coarse quantization. Such problem has been investigated in [3–5]. In terms of signal detection, the work in [6]

proposes an iterative detection and decoding (IDD) scheme, which is based on the exchange of soft information from regular LDPC codes. Moreover, large-scale MIMO with 1-bit ADCs can also be used in millimeter-Wave (mmWave) systems, which are favorable candidates for 5G cellular systems, since they can achieve much larger bandwidths compared to the sub-6 GHz. The authors in [7–9] have discussed channel estimation, signal detection and precoding techniques in such systems.

In this context, oversampling is a technique in which the received signal is sampled at a rate faster than the Nyquist rate. Recently, it has drawn much attention, since it can mitigate the performance loss caused by the coarse quantization. The works in [10–12] have employed oversampling in 1-bit MIMO systems and investigated its advantages in channel estimation and achievable rate. However, the extra samples resulting from oversampling may increase the signal processing cost at the baseband. To reduce such cost, a sliding window based linear detector is proposed in [13], which performs signal detection in a short window rather than a whole block.

In this paper, we propose a dynamic oversampling technique for asynchronous large-scale MIMO systems with 1-bit ADCs at the receiver. Different from prior works, we construct an asynchronous 1-bit oversampled MIMO system and propose a special oversampling technique, named dynamic oversampling, that results in performance gains with small number of processing samples. Two sampling rates are introduced, the initial and the processing sampling rates. The system is initially oversampled at a higher rate and after the sample reduction operation only few weighted data samples are processed for the further operation, where the dimension reduction algorithm is based on the generalized eigenvalue decomposition (GEVD). In the numerical results, we investigate the benefits in terms of the lower bounds on the sum rate capacity, which is calculated through the Bussgang decomposition. We compare the performance of the system with dynamic and standard uniform oversampling techniques.

The rest of this paper is organized as follows: section 2 shows the asynchronous system model of 1-bit oversampled MIMO. Section 3 gives the derivation of the Bussgang-based

lower bound of sum rate capacity and illustrates the proposed GEVD based dynamic oversampling technique. In section 4, the simulation results are presented and section 5 concludes the paper.

The following notations are used throughout the paper: matrices are in bold capital letters while vectors in bold lowercase. \mathbf{I}_n denotes an $n \times n$ identity matrix. Additionally, $\text{diag}(\mathbf{A})$ is a diagonal matrix only containing the diagonal elements of \mathbf{A} and $\text{blkdiag}(\cdot)$ is a block-diagonal operation. The vector or matrix transpose and conjugate transpose are represented by $(\cdot)^T$ and $(\cdot)^H$, respectively. \otimes is the Kronecker product and $\det(\cdot)$ is the determinant function. The convolution of f and g is written as $f \star g$.

2. SYSTEM MODEL

A single-cell multi-user large-scale multiple-antenna scenario is considered in Fig. 1. In the network considered, N_t single-antenna terminals simultaneously transmit signals to the BS equipped with N_r receive antennas, where $N_r \gg N_t$. The filtered oversampled signal \mathbf{y} can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (1)$$

where \mathbf{x} is the $NN_t \times 1$ column vector transmitted by N_t terminals for a block of symbols with length N . The vector \mathbf{x} is arranged as follows:

$$\mathbf{x} = [x_{1,1} \quad \cdots \quad x_{N,1} \quad x_{1,2} \quad \cdots \quad x_{N,N_t}]^T, \quad (2)$$

where $x_{i,j}$ corresponds to the transmitted symbol of terminal j at time instant i . Each symbol is independent identically distributed (IID) and has unit power so that $E[|x_{i,j}|^2] = 1$. Furthermore, \mathbf{n} is the filtered oversampled noise vector of size $MN_rN \times 1$ expressed by

$$\mathbf{n} = (\mathbf{I}_{N_r} \otimes \mathbf{G}) \mathbf{w}, \quad (3)$$

where the noise matrix $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}_{3MN_rN}, \sigma_n^2 \mathbf{I}_{3MN_rN})$ contains IID complex Gaussian random variables with zero mean and variance σ_n^2 . \mathbf{G} is a Toeplitz matrix described by (4), which contains the coefficients of the matched filter $m(t)$ at different time instants. Note that the noise samples are described such that each entry of \mathbf{n} has the same statistical properties. Since the receive filter has a length of $2MN + 1$ samples, $3MN$ unfiltered noise samples in the noise vector \mathbf{w} need to be considered for the description of an interval of MN samples of the filtered noise \mathbf{n} . T is the symbol period. M and M' denote the initial and processing oversampling rate, respectively ($M > M'$). The equivalent channel matrix \mathbf{H} is described as

$$\mathbf{H} = (\mathbf{H}' \otimes \mathbf{I}_{MN}) \text{blkdiag}([\mathbf{Z}_1, \dots, \mathbf{Z}_{N_t}]) (\mathbf{I}_{NN_t} \otimes \mathbf{u}), \quad (5)$$

where \mathbf{H}' is an $N_r \times N_t$ matrix whose element in the i th row and j th column corresponds to the channel coefficient

between terminal j and receive antenna i . The vector \mathbf{u} is employed as an oversampling operator defined as the vector with the size $M \times 1$

$$\mathbf{u} = [0 \quad \cdots \quad 0 \quad 1]^T. \quad (6)$$

The matrix $\mathbf{Z}_{n_t} \in \mathbb{R}^{MN \times MN}$ is a Toeplitz matrix that contains the coefficients of $z(t) = p(t) \star m(t)$ at different time instants, and is given by

$$\mathbf{Z}_{n_t} = \begin{bmatrix} z[n_d] & z[n_d + \frac{T}{M}] & \cdots & z[n_d + NT - \frac{1}{M}T] \\ z[n_d - \frac{T}{M}] & z[n_d] & \cdots & z[n_d + NT - \frac{2}{M}T] \\ \vdots & \vdots & \ddots & \vdots \\ z[n_d - NT + \frac{1}{M}T] & z[n_d - NT + \frac{2}{M}T] & \cdots & z[n_d] \end{bmatrix}. \quad (7)$$

In the asynchronous system, due to the different transmit time to the BS each terminal has its own time delay n_d resulting in different \mathbf{Z}_{n_t} .

Let $\mathcal{Q}(\cdot)$ represent the 1-bit quantization at the receiver, the resulting quantized signal $\mathbf{y}_\mathcal{Q}$ is

$$\mathbf{y}_\mathcal{Q} = \mathcal{Q}(\mathbf{y}) = \mathcal{Q}(\Re\{\mathbf{y}\}) + j\mathcal{Q}(\Im\{\mathbf{y}\}), \quad (8)$$

where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ get the real and imaginary part, respectively. They are element-wisely quantized to $\{\pm 1\}$ and scaled to $\{\pm \frac{1}{\sqrt{2}}\}$ based on the sign.

3. CAPACITY LOWER BOUND AND DIMENSION REDUCTION

Similar to the work in [14], we derive the capacity lower bound of the 1-bit MIMO systems through the Bussgang decomposition. Based on the derived bound, we propose a dynamic oversampling technique to obtain a small number of samples, which largely contribute to the sum rate, from all the received quantized samples.

3.1. Bussgang based Sum Rate Capacity

According to the Bussgang's theorem [15], (8) can be decomposed as

$$\mathbf{y}_\mathcal{Q} = \mathbf{A}\mathbf{y} + \mathbf{n}_q = \mathbf{A}\mathbf{H}\mathbf{x} + \mathbf{A}\mathbf{n} + \mathbf{n}_q. \quad (9)$$

The vector \mathbf{n}_q is the statistically equivalent quantization noise with covariance matrix $\mathbf{C}_{\mathbf{n}_q} = \mathbf{C}_{\mathbf{y}_\mathcal{Q}} - \mathbf{A}\mathbf{C}_{\mathbf{y}}\mathbf{A}^H$, where $\mathbf{C}_{\mathbf{y}_\mathcal{Q}}$ is obtained through the arcsin law [16]

$$\mathbf{C}_{\mathbf{y}_\mathcal{Q}} = \frac{2}{\pi} (\sin^{-1}(\mathbf{K}\mathbf{C}_{\mathbf{y}}^R\mathbf{K}) + j\sin^{-1}(\mathbf{K}\mathbf{C}_{\mathbf{y}}^I\mathbf{K})) \quad (10)$$

and

$$\mathbf{C}_{\mathbf{y}} = \mathbf{H}\mathbf{H}^H + \sigma_n^2(\mathbf{I}_{N_r} \otimes \mathbf{G}\mathbf{G}^H). \quad (11)$$

The matrix $\mathbf{A} \in \mathbb{R}^{MN_r \times MN_r}$ is the linear operator chosen independently from \mathbf{y} and is given by

$$\mathbf{A} = \mathbf{C}_{\mathbf{y}_\mathcal{Q}}^H \mathbf{C}_{\mathbf{y}}^{-1} = \sqrt{\frac{2}{\pi}} \mathbf{K}, \quad \text{with } \mathbf{K} = \text{diag}(\mathbf{C}_{\mathbf{y}})^{-\frac{1}{2}}, \quad (12)$$

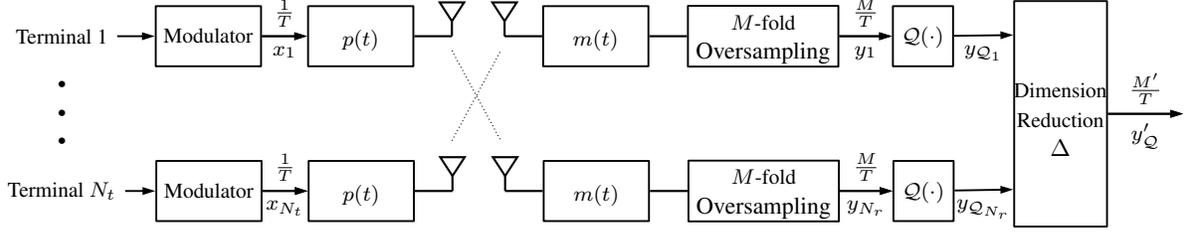


Fig. 1: System model of multi-user multiple-antenna system with 1-bit ADCs and reduced-rank based oversampling at the receiver

$$\mathbf{G} = \begin{bmatrix} m(-NT) & m(-NT + \frac{1}{M}T) & \dots & m(NT) & 0 & \dots & 0 \\ 0 & m(-NT) & \dots & m(NT - \frac{1}{M}T) & m(NT) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & m(-NT) & m(-NT + \frac{1}{M}T) & \dots & m(NT) \end{bmatrix}_{MN \times 3MN} \quad (4)$$

where $\mathbf{C}_{\mathbf{y}\mathbf{y}_Q}$ denotes the cross-correlation matrix between the received signal \mathbf{y} and its quantized signal \mathbf{y}_Q , described by

$$\mathbf{C}_{\mathbf{y}\mathbf{y}_Q} = \sqrt{\frac{2}{\pi}} \mathbf{K} \mathbf{C}_{\mathbf{y}}. \quad (13)$$

Letting $\mathbf{n}' = \mathbf{A}\mathbf{n} + \mathbf{n}_q$ and assuming \mathbf{n}' is Gaussian distributed, which minimizes the mutual information, the sum rate capacity lower bound is calculated as

$$\begin{aligned} C &= \frac{1}{N} \log_2 \det(\mathbf{C}_{\mathbf{y}_Q} \mathbf{C}_{\mathbf{n}'}^{-1}) \\ &= \frac{1}{N} \log_2 \det(\mathbf{I} + \mathbf{A} \mathbf{H} \mathbf{H}^H \mathbf{A} (\mathbf{A} \mathbf{C}_{\mathbf{n}} \mathbf{A} + \mathbf{C}_{\mathbf{n}_q})^{-1}). \end{aligned} \quad (14)$$

3.2. Dimension Reduction

As illustrated in the previous section, the system is initially oversampled at a higher rate M . In order to maintain low complexity for the further signal processing, only few samples are combined and selected based on the reduced-rank GEVD algorithm.

Assuming the dimension reduction operation can be mathematically described as a linear transformation with the matrix Δ [17], the reduced received signal is

$$\mathbf{y}'_Q = \Delta \mathbf{y}_Q, \quad (15)$$

where Δ is a matrix with size of $M'NN_r \times MNN_r$. According to (14), the modified capacity lower bound is then

$$C = \frac{1}{N} \log_2 \det(\Delta \mathbf{C}_{\mathbf{y}_Q} \Delta^H (\Delta \mathbf{C}_{\mathbf{n}'} \Delta^H)^{-1}). \quad (16)$$

The optimization problem that corresponds to the design of the optimal Δ_{opt} that can obtain the highest achievable sum rates is described as

$$\Delta_{\text{opt}} = \arg \max_{\Delta} \log_2 \det(\Delta \mathbf{C}_{\mathbf{y}_Q} \Delta^H (\Delta \mathbf{C}_{\mathbf{n}'} \Delta^H)^{-1}). \quad (17)$$

Since the determinant is a log-concave function [18] and with the properties of the determinant, (17) is equivalent to

$$\Delta_{\text{opt}} = \arg \max_{\Delta} \frac{\det(\Delta \mathbf{C}_{\mathbf{y}_Q} \Delta^H)}{\det(\Delta \mathbf{C}_{\mathbf{n}'} \Delta^H)}. \quad (18)$$

According to [19], the problem (18) can be efficiently solved by the GEVD algorithm, which is summarized in Algorithm 1. In steps 5 and 6, the matrix Λ is a diagonal matrix containing the eigenvalues and the corresponding eigenvectors are placed in the matrix Φ . Since the largest eigenvalues contains more useful information, the eigenvectors corresponding to these dominant eigenvalues are used to form the matrix Δ_{opt}^H .

Algorithm 1 GEVD algorithm

- 1: Eigenvalue decomposition: $\Phi_B, \Lambda_B \leftarrow \mathbf{C}_{\mathbf{n}'} \Phi_B = \Phi_B \Lambda_B$
 - 2: $\tilde{\Phi}_B \leftarrow \tilde{\Phi}_B = \Phi_B \Lambda_B^{-\frac{1}{2}}$
 - 3: $\tilde{\mathbf{A}} \leftarrow \tilde{\mathbf{A}} = \tilde{\Phi}_B^H \mathbf{C}_{\mathbf{y}_Q} \tilde{\Phi}_B$
 - 4: Eigenvalue decomposition: $\Phi_A, \Lambda_A \leftarrow \tilde{\mathbf{A}} \Phi_A = \Phi_A \Lambda_A$
 - 5: $\Lambda \leftarrow \Lambda_A$
 - 6: $\Phi \leftarrow \Phi = \tilde{\Phi}_B \Phi_A$
 - 7: Extract the eigenvectors in Φ related to the $M'NN_r$ biggest diagonal values in Λ
 - 8: Construct Δ_{opt}^H column-wisely
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Compared to the uniform oversampled system, which does not need the reduction matrix Δ ¹, the biggest computational cost of the proposed dynamic oversampling technique lies in the eigenvalue decomposition in steps 1 and 4, which is $\mathcal{O}((MNN_r)^3)$.

¹For the uniform oversampled system Δ is a square matrix. The sum rates cannot be maximized, when all the samples are used. For this reason Δ can be neglected.

4. NUMERICAL RESULTS

In this section, we evaluate the proposed dynamic oversampling technique in terms of the Bussgang-based sum rate capacity, which is obtained by averaging over results obtained from 100 independent realizations of the channel matrix \mathbf{H}' in a MIMO system with $N_t = 4$ and $N_r = 64$. The delay of each terminal is uniformly distributed between $-T$ and T . Each transmission block contains 10 symbols and Gaussian signaling is considered. The $m(t)$ and $p(t)$ filters are normalized root-raised-cosine (RRC) filters with a roll-off factor of 0.8. The SNR is defined as $10 \log(\frac{1}{\sigma_n^2})$.

We firstly investigate the receiver power consumption of the dynamic oversampled system versus the number of bits used by the ADCs. Fig. 2 shows the simplified power consumption as a function of the quantization bits. Based on [20], without considering the common parts between systems with an arbitrary number of bits the power consumption at the receiver can be reduced to

$$P_{\text{total}} \approx 2N_r(cP_{\text{AGC}} + P_{\text{ADC}}), \quad (19)$$

where P_{AGC} denotes the power consumption of automatic gain control (AGC). c is chosen as 0 for 1-bit system and 1 for systems with more-bits. The P_{ADC} is calculated as

$$P_{\text{ADC}} = \text{FOM}_w \times M f_n \times 2^b, \quad b = 1, 2, \dots \quad (20)$$

where f_n is the Nyquist-sampling rate and b denotes the quantization bits. From [20], $P_{\text{AGC}} = 2\text{mW}$, FOM_w is 200 fJ/conversion-step at 50 MHz bandwidth and f_n is 100 MHz. With the illustrated figure, we can see that for the system with $M' = 2$ the dynamic oversampling technique $M > M'$ consumes almost the same power as that with the uniform oversampling technique $M = M'$ below 4 bits. This reveals the receiver power consumption advantage of the proposed dynamic oversampling technique.

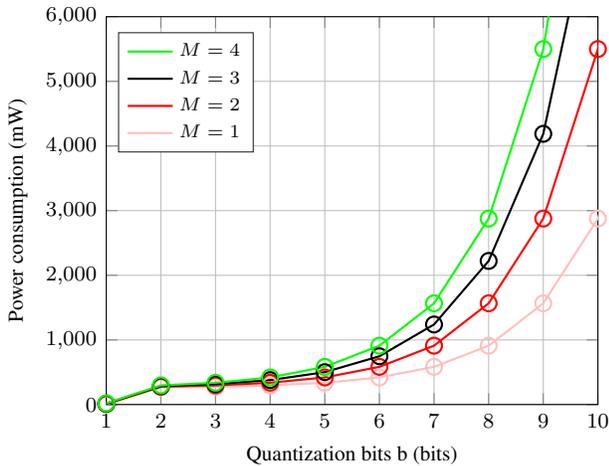


Fig. 2: Simplified power consumption at the receiver

Furthermore, Fig. 3 makes the comparison of capacity lower bounds between dynamic and uniform oversampled systems under the same processing sampling rate $M' = 2$. Note that for the uniform oversampled system all received samples are used for further processing. From the results we can see that the dynamic oversampled system has almost the same capacity lower bound as the uniform oversampled system. This important result indicates that the proposed dynamic oversampling technique can use less processing samples to achieve the same sum rates as compared to the standard uniform oversampling technique. Another observation is that under the same processing sampling rate $M' = 2$ the dynamic oversampled system has a significant performance gain as compared to the uniform oversampled system with $M = 2$, where the gain achieves its saturation after $M = 4$. These observations show the performance advantages of the proposed oversampling technique.

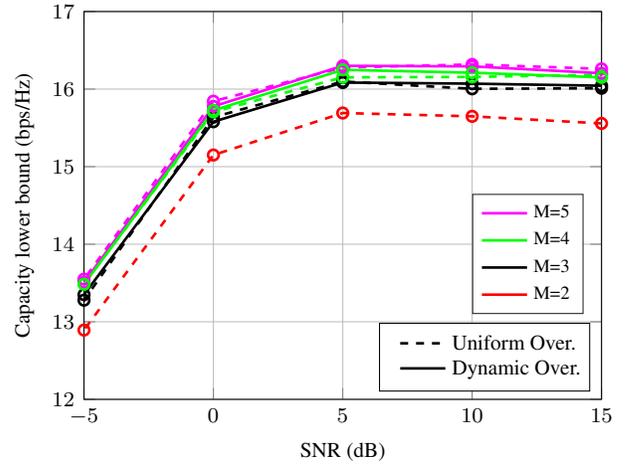


Fig. 3: Comparison of capacity lower bounds between uniform and dynamic oversampled systems with the same $M' = 2$

5. CONCLUSION

This work has proposed a dynamic oversampling technique for asynchronous large-scale MIMO systems with 1-bit quantization at the receiver. We have derived the sum rate lower bound based on the Bussgang decomposition. Simulation results have shown that the proposed oversampling technique can use small number of processing samples to achieve the same capacity lower bound as the standard uniform oversampling technique while maintaining almost the same power consumption. Further, with the same number of processing samples the proposed technique can achieve much higher sum rates. As an extension of the current work, we will investigate the advantage of proposed technique in different system designs, such as minimizing the mean square error of detected symbols.

6. REFERENCES

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